A Theoretical Three-Port Coaxial-Line Rectangular-Waveguide Model and Its Application to Millimeter-Wave Structures

A. Rydberg and B. N. Lyons

Abstract—**A theoretical three-port model for coaxial-line rectangular-waveguide junction is described and evaluated using several different realistic millimeter-wave mount structures. The model is found to be usable with good accuracy over a large variation in mount dimensions. The model is specifically applied in calculations of the embedding impedance seen by the diode in a millimeter-wave frequency multiplier. The three-port model is an improvement over simpler models hitherto used for devices such as millimeter-wave frequency multipliers, in taking into account all parameters in the mount.**

I. Introduction

The coaxial-line rectangular-waveguide junction is an important part in many microwave structures. Such two port circuits have been accurately described by Williamson [1], [2]. However, in many designs, such as the output circuit of a frequency multiplier involving a so called idler resonator, a three-port circuit is formed consisting of 1) the gap in the post formed by the device, 2) the idler resonator, and 3) the coaxial-line input port for the pump frequency (see Fig. 1).

By extending the theoretical models derived by Williamson [1], [2] for two-port coaxial-line rectangular-waveguide circuits to a three-port circuit we have been able to investigate the influence of the mount dimensions on the embedding impedance seen by the device positioned in the gap of the post (see Fig. 1) in realistic millimeter-wave structures.

II. Theory

Three port junctions such as the one in Fig. 1 may be represented by a three-port network problem defined by three equations:

$$I_1 = Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3, \quad (1)$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3, \quad (2)$$

$$I_3 = Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3, \quad (3)$$

where the indexes refer to the port number (see Fig. 1).

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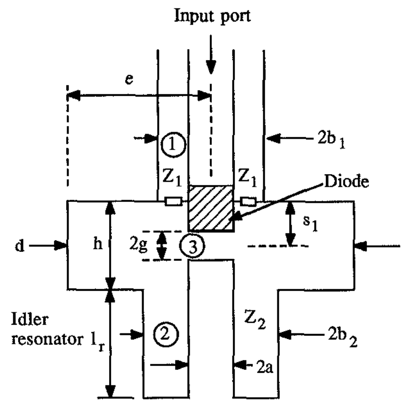

Fig. 1. Cross-section view of a three-port coaxial-line rectangular-waveguide mount structure. Z_1 is the impedance seen at the end of coaxial line 1. Z_2 is the characteristic TEM-mode impedance of coaxial-line 2. h = 36mm, d = 72mm.

Solving (1)–(3) for the input admittance $Y_3, (= I_3/V_3)$, seen by the device gives the following expression for Y_3:

$$Y_3 = Y_{33} + \big(2Y_{12}Y_{13}Y_{23} - Y_{13}^2(Y_2 + Y_{22}) - Y_{23}^2(Y_1 + Y_{11})\big) / \cdot \big((Y_1 + Y_{11})(Y_2 + Y_{22}) - Y_{12}^2\big). \quad (4)$$

The values of the elements in (4) can be derived from Williamson's results on two-port coaxial-line rectangular-waveguide structures [1], [2], thus the admittance elements in (1)–(3) are functions of the mount dimensions in the following way:

$$Y_{11} = f(d, h, a, b_1), \quad Y_{22} = f(d, h, a, b_2),$$

$$Y_{33} = f(d, h, a, g, s_1), \quad Y_{12} = f(d, h, a, b_1, b_2),$$

$$Y_{13} = f(d, h, a, b_1, g, s_1),$$

$$Y_{23} = f(d, h, a, b_2, g, h\text{-}s_1).$$

The parameters are defined in Fig. 1. Here, we assume $Y_{21} = Y_{12}$, $Y_{31} = Y_{13}$, and $Y_{32} = Y_{23}$, by reciprocity.

Fig. 2 shows the equivalent circuit for the three port assuming that the only propagating mode in the waveguide is the TE_{10}-mode. The values for B_a, B_b, B_c, B_d, B_e, and B_f as well as R_1, R_2, and R_3 can be deduced from [1], [2] and are listed in [3].

The theoretical three-port model, here called "three-port," is compared with a simplified model, here called the "sim-

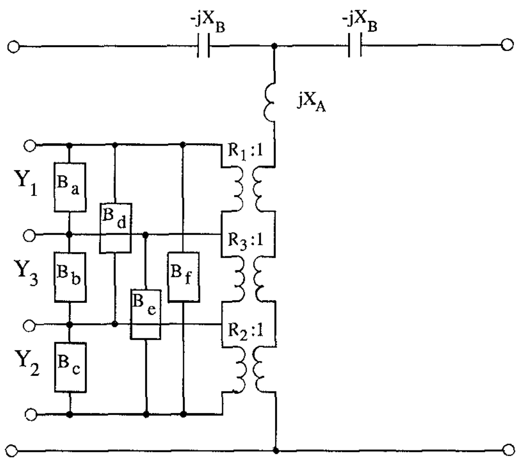

Fig. 2. Equivalent circuit for the mounting structure assuming that the only propagating mode in the waveguide is the TE_{10}-mode. Y_1 and Y_2 are the input admittances of coaxial port 1 and 2, respectively [3]. For X_A and X_B (see [1], [2]).

ple" model. The "simple" model assumes that the embedding impedance seen by the device is equal to the sum of the waveguide impedance and the impedances of the coaxial lines, No. 1 and 2 in Fig. 1 [4]. However, the "simple" model that has been used with fairly good results in designing millimeter-wave multipliers does not fully explain the influence of the mount dimensions on the device embedding impedance [5], since none of the parameters s_1, g or a, b, for constant coaxial impedance, is taken into account in this model.

III. On the Validity of the Model

Using a waveguide WR-284 mount, with a design similar to Fig. 1, the embedding impedance seen by the post-gap was calculated using the "three-port" model, and measured using a coaxial-line probe placed in the post-gap, according to [6], for a number of different mount dimensions. Some of the calculated and measured embedding impedances for mounts which emulate realistic millimeter-wave waveguide mounts are shown in Figs. 3(a)–(f) [3]. A movable waveguide backshort was incorporated in the mount, and was varied over half a guide wavelength in the calculations and measurements. In calculating the impedance transformation between the coaxial-line probe and the radial-line formed in the post-gap, the theory in [7] was used. It can be seen in Fig. 3 that the theoretical and experimental results agree fairly well, taking into account the large post diameter and post-gap dimensions investigated, being 48% and 70% of the full waveguide width and height respectively in Fig. 3(d).

IV. Calculations and Discussions

Using the "three-port" model and the "simple" model calculations are made on the output circuit of a 250 GHz frequency tripler, where the idler resonator is formed by the coaxial-line 2 in Fig. 1, having a length l_r of $\lambda/2$ at 250 GHz.

Here, we have assumed that the diode chip forms a part of the post (see Fig. 1) and is contacted using a whisker wire that is placed in the gap of the post. The whisker inductance is calculated using the formula derived by Tolmunen and Räisänen [8]. It is also assumed that no higher order mode can be excited in the coaxial lines.

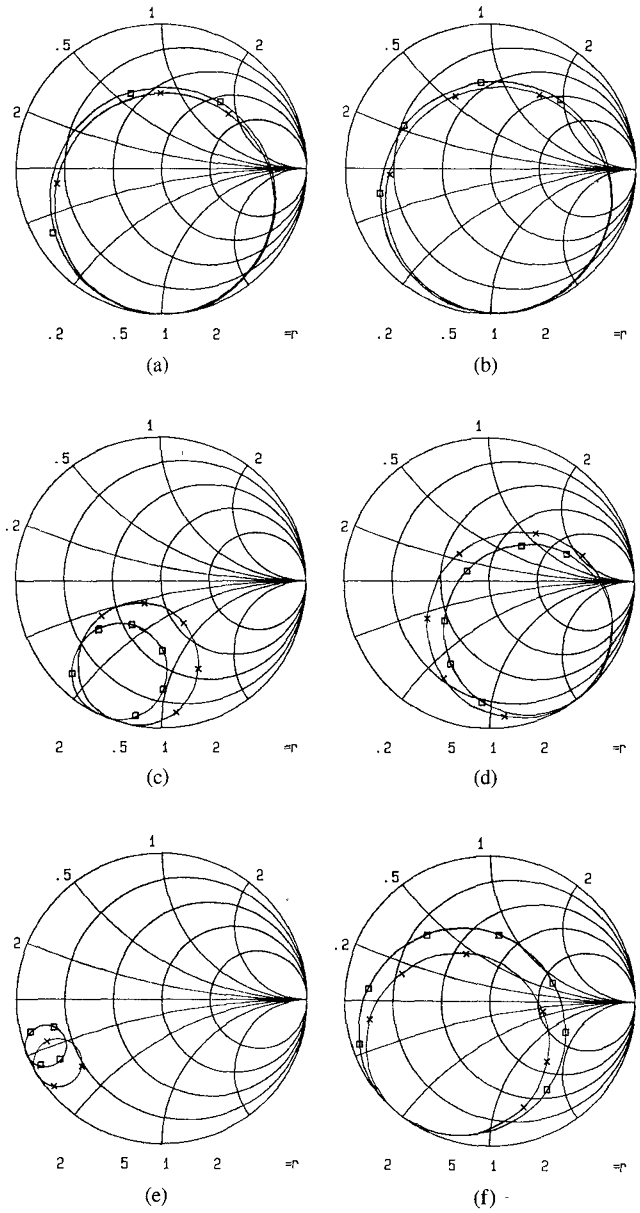

Fig. 3. SMITH-charts showing the measured and calculated embedding impedances seen by the gap in the post (see Fig. 1). (□): Measured impedance. (x): Calculated impedance. $Z_1 = 0$ ohm. $e/d = 0.5$ (For Z_1 and mount dimensions see Fig. 1.) (a) Frequency = 3 GHz, $a/d = 0.146$, $g/h = 0.355$, $l_2 = 0$ mm. (b) Frequency = 3 GHz, $a/d = 0.146$, $g/h = 0.473$, $l_2 = 0$ mm. (c) Frequency = 3 GHz, $a/d = 0.242$, $g/h = 0.117$, $l_2 = 0$ mm. (d) Frequency = 3 GHz, $a/d = 0.242$, $g/h = 0.355$, $l_2 = 0$ mm. (e) Frequency = 4 GHz, $a/d = 0.146$, $g/h = 0.117$, $l_2 = 0$ mm. (f) Frequency = 4 GHz, $a/d = 0.146$, $g/h = 0.117$, $l_2 = 10$ mm.

The backshort in the input, not shown in Fig. 1, and the output waveguides are varied over $\lambda g/2$ in the calculations.

Using the "three-port" model and the "simple" model the embedding impedance seen by the device at the input frequency was calculated for three different parameters in Fig. 1 that is s_1/h, g/h and the TEM-mode impedance Z_2 of the coaxial-line 2 (see Fig. 4).

The large increase in embedding reactance with decreasing s_1/h (see Fig. 4(a)) is due to the increase in length of the

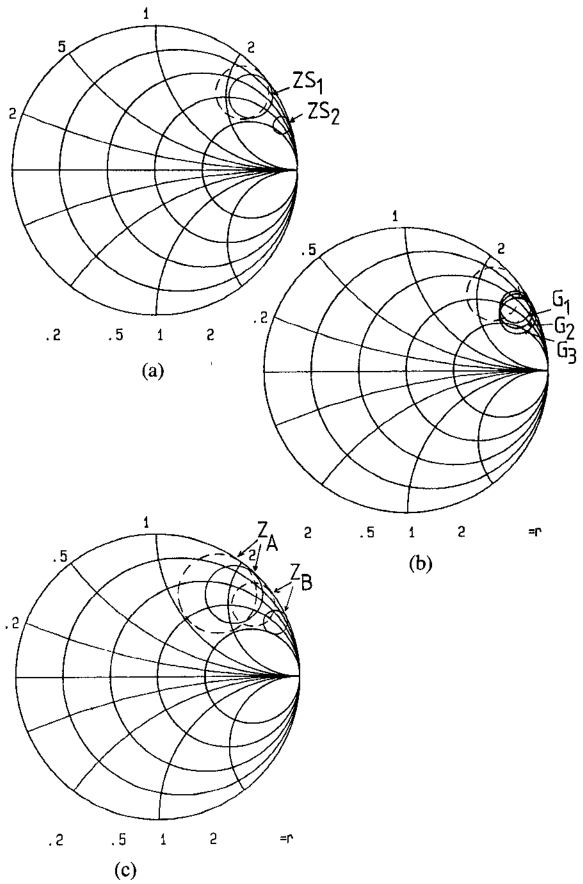

Fig. 4. Embedding impedance at the input frequency, 83.3 GHz, for the tripler as a function of s_1/h, g/h, and Z_2 (see Fig. 1). $d = 0.77$ mm, $h = 0.19$ mm, $a/d = 0.115$, $b_1/d = 0.183$, $e/d = 0.5$. Z_1 varies as a function of the input backshort, not shown in Fig. 1. Whisker is assumed to be 150 μm long, having a diameter of 10 μm. (- -): Calculations using "simple" model. (—): Calculations using "three port" model. (a) ZS_1 is $s_1/h = 0.85$, ZS_2 is $s_1/h = 0.15$. $b_2/d = 0.265$. $g/h = 0.15$. (b) G_1 is $g/h = 0.15$, G_2 is $g/h = 0.075$, G_3 is $g/h = 0.0375$. $b_2/d = 0.265$. $s_1/h = 0.5$. (c) $Z_A$is $Z_2 = 30$ ohm ($b_2/d = 0.19$), Z_B is $Z_2 = 60$ ohm ($b_2/d = 0.313$). $s_1/h = 0.5$. $g/h = 0.15$.

part of the post linked to the coaxial idler resonator (see Fig. 1). The total length of the resonator, l_r in Fig. 1, plus $(h - (s_1 + g))$ in Fig. 1 becomes close to $\lambda/4$ at the input frequency. Thus the large increase in reactance decreasing s_1 shifts the input matching to lower frequencies, a result that could not have been anticipated using the "simple" model that is not affected by the variation in s_1.

The embedding impedance is found to be less affected by the gap width, g in Fig. 1, except for very small gaps (see Fig. 4(b)).

The influence on the embedding impedance by the idler impedance Z_2 is greater using the "three-port" model compared to the "simple" one (see Fig. 4(c)). This is due to the "three-port" model being affected by the absolute value of (a) and (b) and not only by the idler impedance Z_2 as for the "simple" one.

V. Conclusion

A theoretical three-port model for coaxial-line rectangular-waveguide junctions has been derived which more accurately describes the influence of the output circuit of a millimeter-wave frequency tripler on the embedding impedance seen by the device than models hitherto used. The model has been found to be usable for realistic millimeter-wave mounting structures, having post diameters and post-gap dimensions of at least 48% and 70% of the waveguide width and height, respectively.

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